Kernel Search: a simple heuristic framework for MILP problems

Renata Mansini

Department of Information Engineering, University of Brescia
renata.mansini@unibs.it

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Outline

- Basic Kernel Search (KS)
  - Heuristic Structure
  - KS Phases
  - Literature
  - Critical Aspects
- Enhanced Iterative KS (EIKS)
  - Innovative features
- Application to MCTOP
  - The problem
  - EIKS
  - Primal KS (PKS)
  - Branch-and-Cut algorithm
  - Computational results
- Conclusions

Mixed Integer Linear Problem (MIP)

$$\max \ \mathbf{c}^{\mathbf{T}}\mathbf{x}$$

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$

$$x_j \geq 0 \quad j \in \mathcal{C}$$

$$x_j \in \{0,1\} \quad j \in \mathcal{B}$$

$$x_j \geq 0 \quad \text{integer } j \in \mathcal{I}$$

- When $\mathcal{I} = \emptyset$ we have 0–1 MIP

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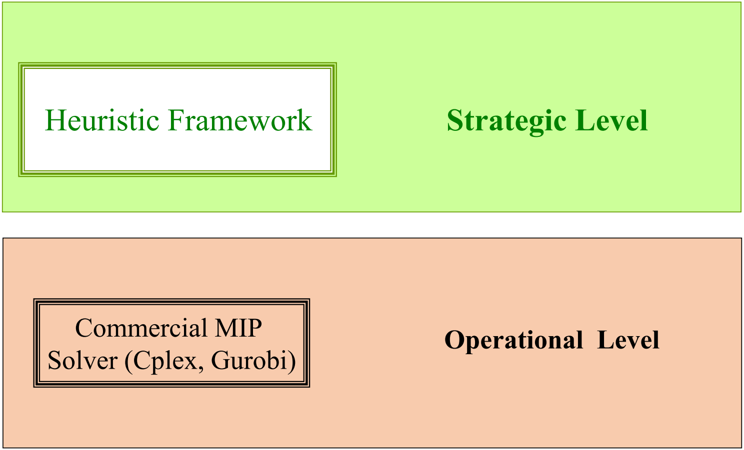
- When $\mathcal{I} = \emptyset$ we have 0–1 MIP
- Binary variables model decisions of selecting/refusing items

Solution Approach

When we first introduced KS we looked for:

- ✓ a general-purpose method
- ✓ a method able to find high quality solutions in reasonable time
- ✓ a method that requires very little implementation effort

Kernel Search: Main Structure



Kernel Search: Main Idea

Identify a KERNEL SET

$\Downarrow$

a RESTRICTED set of promising variables
(more probably selected in an optimal solution)

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$\Downarrow$

a RESTRICTED set of promising variables
(more probably selected in an optimal solution)

RESTRICTED:

- not too large (avoid computational effort)
- not too small (allow correlated items to be jointly selected)

Kernel Search: the Heuristic Framework

PHASE 1
(Construction)

INITIAL KERNEL SET

Select promising variables in the Kernel Set

INITIAL SOLUTION

Solve a restricted MIP on the Kernel Set

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PHASE 2
(Improvement)

SOLVE SUBPROBLEMS

Solve a sequence of restricted subproblems

KERNEL SET UPDATE

Iteratively extend the Kernel Set

Kernel Search: PHASE 1 (Construction)

1. Kernel Set Construction:

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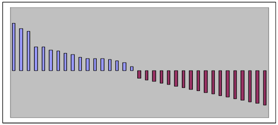
   a) Sort variables according to a predefined criterion (LP values, reduced costs)



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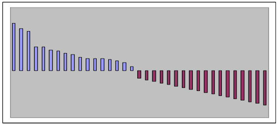
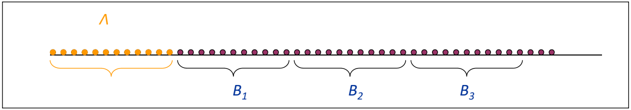


   b) Identify the Kernel Set (Λ) and partition remaining variables into buckets



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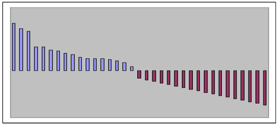

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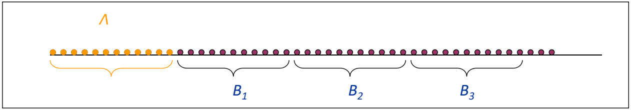
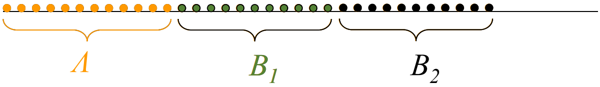


2. Initial Solution: solve subproblem MIP(Λ) by Gurobi/Cplex

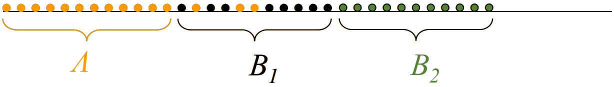
Kernel Search: PHASE 2 (Improvement)

Repeat until no more buckets are available:

1. Construct subproblem MIP($\Lambda \cup B$) on current bucket B:



2. Solve subproblem MIP($\Lambda \cup B$):
   - update lower bound (incumbent integer solution max problem)
   - update Kernel Set Λ



Kernel Search: Literature review

Basic Version:

- Introduced in a simplified version in Mansini [1997]
- Angelelli, Mansini, Speranza [2010]: Multi-dimensional Knapsack Problem
- Angelelli, Mansini, Speranza [2012]: Portfolio Selection Problem
- Guastaroba et al. [2012a], [2012b], [2014]: Index Tracking Problem, Facility Location Problem

New Variants

- Adaptive Kernel Search (Guastaroba, Savelsbergh, Speranza [2017])
- Enhanced Iterative Kernel Search (Hanafi, Mansini, Zanotti [2018])

Kernel Search: overcoming drawbacks

Critical aspects:

1. Find alternatives to reduced costs when they do not provide a clear signal on promising variables
2. Find an effective way for variables to enter/leave the Kernel Set (manage effectively the trade-off between solution quality and computational effort)
3. Identify a way to measure correlation among variables
4. Optimize the breakdown of the global solution time used

Adaptive Kernel Search
(Guastaroba, Savelsbergh, Speranza [2017])

Enhanced Iterative Kernel Search (EIKS)

Iterative variant: scroll the buckets more than once (bucket iterations)

Innovative aspects:

1. Different sorting rules that can be interchanged in bucket iterations
2. Dynamic procedure to control Kernel Set and buckets size
3. Affinity as new paradigm for measuring correlation among variables
4. Dynamic adjustment of subproblems solution time

Multi-visit Clustered Team Orienteering Problem

Given:

- a set of nodes $V = N \cup \{0, n+1\}$, where $N = \{1, ..., n\}$ (set of tasks) partitioned into m clusters (customers) $N = \bigcup_{h=1}^{m} C_h$
- a set of arcs $A = \{(i,j) : i \neq n+1; j \neq 0; i \neq j; i,j \in V\}$
- a set K of dedicated vehicles (technicians), each one specialized in the execution of a subset of nodes (tasks)
- a time bound T_{max} for each vehicle
- a profit p_h for each cluster $h = 1, ..., m$
- a service time d_i for each $i \in N$
- a traveling time $t_{ij}, (i,j) \in A$

Multi-visit Clustered Team Orienteering Problem

Constraints:

- each route starts from the depot at time 0
- the traveling time plus the service time of each route cannot exceed T_{max}
- if a cluster is selected all its nodes have to be executed according to a predefined order
- each node of a selected cluster has to be visited exactly once by one vehicle enabled to serve it
- no waiting time allowed to a node

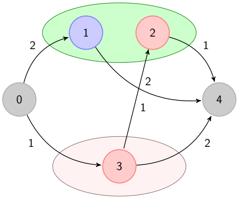
Objective: Find a set of routes, each one not exceeding T_{max}, that maximizes total collected profit while satisfying precedence constraints

Real Case Application

Large company assembling equipments and furniture on site. Tasks are accomplished by technicians working in team.

Request: Evaluate the advantage to let technicians move independently among customers

Assumption: No waiting times at nodes



2 vehicles, 2 clusters, 3 nodes (service time $d_1 = 2; d_2 = 1; d_3 = 1$), $T_{max} = 6$

No optimal solution by forcing the use of two vehicles.

Problem Formulation 1/2

$$max \sum_{h=1}^{m} p_h y_h \tag{1}$$

$$\sum_{(0,i)\in\delta^+(0)} x_{0i}^k = \sum_{(i,n+1)\in\delta^-(n+1)} x_{i,n+1}^k = 1 \qquad k \in K \tag{2}$$

$$\sum_{(i,j)\in\delta^+(i)} x_{ij}^k = \sum_{(j,i)\in\delta^-(i)} x_{ji}^k = u_i^k \quad i \in N, k \in K_i \tag{3}$$

$$\sum_{k\in K_i} u_i^k = y_h \quad i \in C_h, h = 1, ..., m, \tag{4}$$

$$x_{ij}^k \in \{0,1\} \quad (i,j) \in A, k \in K \tag{5}$$

$$u_i^k \in \{0,1\} \quad i \in N, k \in K_i \tag{6}$$

$$y_h \in \{0,1\} \quad h = 1, ..., m \tag{7}$$

Problem Formulation 2/2

$$\sum_{(i,j)\in\delta^+(i)} z_{ij} - \sum_{(j,i)\in\delta^-(i)} z_{ji} = \sum_{k\in K}\sum_{(i,j)\in\delta^+(i)} (t_{ij}+d_i)x^k_{ij} \quad i \in N \qquad (8)$$

$$z_{0i} = t_{0i}\sum_{k\in K} x^k_{0i} \quad i \in N \qquad (9)$$

$$z_{ij} \geq (t_{0i}+t_{ij}+d_i)\sum_{k\in K} x^k_{ij} \quad (i,j)\in A\setminus\{(0,n+1)\} \qquad (10)$$

$$z_{ij} \leq (T_{max}-d_j-t_{j,n+1})\sum_{k\in K} x^k_{ij} \quad (i,j)\in A\setminus\{(0,n+1)\} \qquad (11)$$

$$\sum_{(s,i)\in\delta^-(i)} z_{si} + d_i\sum_{k\in K} u^k_i \leq \sum_{(s,j)\in\delta^-(j)} z_{sj} \quad i\in\bar{C}_h, j=i+1, h=1,\ldots,m \qquad (12)$$

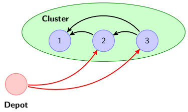
$$z_{ij} \geq 0 \quad (i,j)\in A\setminus\{(0,n+1)\} \qquad (13)$$

Maffioli, Sciomachen [1997]: A MIP model for solving ordering problems

Bianchessi, Mansini, Speranza [2018] (Team Orienteering Problem)

Model Enforcement: Variables Bound

- Variables fixing:
  - $x_{0j}^{k} = 0 \quad j \in N_k : \Gamma_j^- \neq \emptyset$
  - $x_{ij}^{k} = 0 \quad i, j \in N_k, j \in \Gamma_i^-, k \in K_i \cap K_j$



- Upper and lower bounds enforcement of variables:

$$z_{ij} \leq (T_{max} - d_j - t_{j,n+1} - \sum_{u \in \Gamma_j^+} d_u) \sum_{k \in K} x_{ij}^k$$

$$z_{ij} \geq \max\{t_{0i} + t_{ij} + d_i + \sum_{u \in \Gamma_i^-} d_u; \;\; t_{0j} + \sum_{u \in \Gamma_j^-} d_u\} \sum_{k \in K} x_{ij}^k$$

Model Enforcement: Valid Inequalities

- Global Time Cuts:

  $\sum_{(i,j)\in A}(t_{ij}+d_j)x_{ij}^k \leq T_{max}$ for each vehicle $k \in K$

  Separation: Added from scratch.

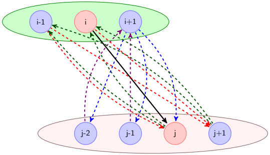
- Connectivity Cuts (CC):

  $\sum_{(i,j)\in\delta^+(S)} x_{ij}^k \geq u_r^k \quad S \subseteq N, r \in S, k \in K$

  Separation: minimum cut problems solved with GUROBI

- Sequentiality Cuts (SC):

$$\sum_{t\in C_h, t<i}\sum_{u\in C_l, u\geq j} x_{tu}^k + \sum_{u\in C_l, u>j} x_{iu}^k +$$
$$\sum_{t\in C_h, t>i}\sum_{u\in C_l, u\leq j} x_{tu}^k +$$
$$\sum_{t\in C_l, t\geq j}\sum_{u\in C_h, u\leq i} x_{tu}^k +$$
$$\sum_{t\in C_l, t<j}\sum_{u\in C_h, u>i} x_{tu}^k \leq$$
$$2\min\{|C_h|;|C_l|\}(1-x_{ij}^k)$$

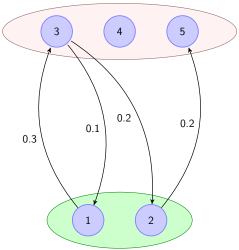


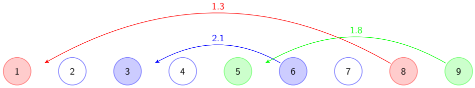
Enhanced Iterative KS: Sorting Rules

1. Compute for each cluster h (max number of iterations $iter$):

$$score[h] \leftarrow score[h] + \chi_1 \frac{p_h}{maxProfit} + \chi_2 \frac{maxSize - |C_h|}{maxSize}$$

2. Sort clusters in non-increasing order of their score
3. Compute *affinity level* for each pair of clusters and adjust sorting:

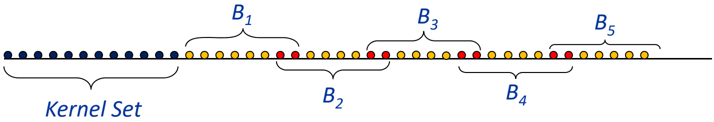




Buckets and Kernel Set Construction

At the beginning of each bucket iteration:

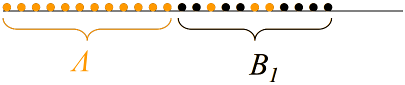
- Kernel Set encompasses all clusters of the best integer solution found so far plus the first ξ clusters in the new sorting
- Remaining clusters are divided into buckets of equal size (but reduced with respect to previous iteration) that partially overlap:



Varying Kernel Set

– Insert procedure:

- $\text{SolveSubproblem}(\Lambda \cup B)$ for each $B \in \mathcal{B}$
- Add to Λ all clusters selected in the subproblem solution



– Eject procedure:

- If $e_h - s_h \geq threshold$, remove cluster C_h from Λ

$\mathcal{S}_h = \{$set of solutions found by KS since cluster C_h entry in $\Lambda\}$

s_h (e_h) number of times C_h belongs (does not) to solutions in $\mathcal{S}_h$

Dynamic Adjustment of Time

For each iteration:

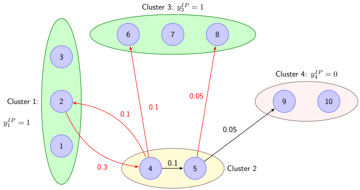
- Kernel Set: $(1+\beta)$ (Global time iteration)/(#buckets+1)
- Each Bucket: $(1+\beta)$ (Residual time)/(residual #buckets)

Motivation: Kernel Set and initial buckets should contain more promising clusters and thus corresponding subproblems should receive higher computational time

Primal Kernel Search (PKS)

Only Kernel Set + 1 Bucket

- Sorting: exploit information provided by local LP relaxation:
  - Affinity on groups of clusters



- Memorization procedure:
  - load active clusters and solution for each subproblem solved
  - if a subproblem has already been solved in a previous call, skip it and solve the next one

Branch & Cut

Dynamic separation of cuts: implemented rules

- Connectivity Cuts: checked at each node of the search tree (root included). Added for $s^* = argmax_{s \in S}\{\bar{u}_s^k\}$, only if its violation is greater than ϵ_1.
- Sequentiality Cuts: checked at each node of the search tree (root included). Added only the first 3 violated inequalities with violation greater than ϵ_2.

Benchmark Problems

Graphs and profits taken from Chao et al. [1997]:
Sets 4,5,6,7.

For each graph:

- 3 random generations of # tasks and service times for each cluster
- 3 numbers of vehicle ($|K|$ =2,3,4)
- 3 values of T_{max}

Per Class: 27 instances

Total: 108 instances (plus a few real case instances)

Experimental Setting

- HW: 64-bit Windows Personal Computer, 2.93 GHz Intel i7 870 processor, and 8 GB of RAM.
- SW: Java 8, GUROBI 7.5.2: all built-in cuts activated, 8 threads, Barrier Algorithm

Exact Algorithms: 2 hours time limit

- BASIC: Compact formulation solved with GUROBI
- B&C: Branch-and-Cut with dynamic separation of connectivity and sequentiality cuts, bounds enforcement, variables fixing
- B&C-PKS: B&C plus Primal Kernel Search

Heuristic Algorithms:

- Iterative KS 15 min.: 10 (first iteration) + 5 (second iteration)
- Primal KS 1 min.: 30 secs (Kernel Set), 30 secs (first bucket)

Instance	BASIC			B&C					B&C-PKS						
	Obj	Ttb	gap	Obj	Ttb	CC	SC	gap	Obj	Ttb	CC	SC	#PKS	#Imp	gap
p5.2.s0	40	33	1214.0	340	6999	1760	1764	47.3	**465**	2575	395	152	53	5	4.12
p5.2.s0t2	125	7005	366.0	445	4290	1331	804	26.8	**515**	2635	492	225	40	7	4.81
p5.2.s0t3	250	7203	153.0	280	3599	2169	1924	119.0	**465**	4963	1132	931	35	8	31.1
p5.2.s1	255	7203	165.0	445	6791	2054	1425	17.7	**465**	4747	1179	527	21	3	10.6
p5.2.s1t2	245	457	205.0	330	3788	1088	571	76.4	**510**	2742	1195	1251	27	7	13.1
p5.2.s1t3	255	4639	218.0	395	7104	646	220	60.4	**585**	5164	1423	713	31	6	6.88
p5.2.s2	90	5460	510.0	350	4358	1119	877	49.2	**420**	6996	1226	1022	44	9	24.8
p5.2.s2t2	320	6154	91.2	305	3098	1885	1509	88.9	**385**	5379	1233	613	27	4	48.9
p5.2.s2t3	170	7036	284.0	100	2362	1973	2207	535.0	**465**	6779	1131	1118	32	7	34.0
p5.3.s0	60	1530	782.0	250	1264	407	45	72.1	**305**	6955	884	237	25	5	42.4
p5.3.s0t2	55	4158	974.0	285	3320	516	34	71.5	**340**	5230	1855	637	33	6	41.4
p5.3.s0t3	65	5292	895.0	240	6493	2538	954	128.0	**315**	6911	930	297	22	5	70.2
p5.3.s1	220	7173	194.0	**275**	7202	444	21	50.7	215	3019	237	19	14	6	89.7
p5.3.s1t2	235	472	206.0	140	6243	592	34	232.0	**245**	6766	293	29	8	2	85.9
p5.3.s1t3	255	5083	208.0	140	4234	1007	66	266.0	**315**	2749	363	37	10	4	61.9
p5.3.s2	80	53	570.0	305	6797	1710	495	46.9	**350**	4677	329	67	29	8	24.9
p5.3.s2t2	195	487	207.0	280	4342	605	36	75.8	**350**	7204	763	96	22	6	39.7
p5.3.s2t3	200	1784	231.0	280	4584	735	48	93.5	**355**	5311	1581	734	27	6	52.4
p5.4.s0	145	7198	262.0	305	2643	369	54	55.3	**360**	4886	275	34	40	7	31.2
p5.4.s0t2	145	3927	311.0	100	3825	418	36	439.0	**355**	4326	1249	250	34	7	52.5
p5.4.s0t3	180	5319	273.0	125	4133	1154	50	382.0	**360**	6408	164	10	21	8	67.5
p5.4.s1	185	6808	213.0	245	2782	283	14	83.2	**330**	7406	238	18	26	6	36.1
p5.4.s1t2	185	266	255.0	170	3888	412	14	203.0	**275**	3539	950	25	21	3	85.5
p5.4.s1t3	200	1972	266.0	240	7197	516	12	138.0	**280**	1772	682	31	18	4	104.0
p5.4.s2	140	6357	245.0	255	3698	321	21	64.1	**330**	1916	496	42	25	4	24.7
p5.4.s2t2	85	857	544.0	**275**	4737	1334	122	69.5	255	3597	2068	562	20	2	79.2
p5.4.s2t3	135	1132	355.0	230	5703	801	17	122.0	**275**	5717	960	54	21	4	85.5
	167.2	3891.0	377.7	264.1	4647.2	1044.0	495.3	133.8	366.1	4828.5	878.6	360.4	26.9	5.5	46.4

Instance	IKS										B&C+PKS		
	Obj	Ttb	#Sub	avKS	maxKS	avSub	maxSub	SOL	ImpIt1	ImpIt2-3	Obj	Ttb	Δ%
p5.2.s0	455	807	22	17.3	24	23.8	30	17	7	4	**465**	2575	2.2
p5.2.s0t2	485	620	16	15.8	21	22.8	29	17	7	3	**515**	2635	5.8
p5.2.s0t3	**495**	577	16	15.4	19	22.4	27	19	7	1	465	4963	-6.5
p5.2.s1	**480**	485	21	18.0	21	24.8	29	20	8	1	465	4747	-3.2
p5.2.s1t2	465	527	16	15.7	19	22.6	27	19	7	0	**510**	2742	8.8
p5.2.s1t3	555	468	17	18.6	23	25.4	31	23	9	0	**585**	5164	5.1
p5.2.s2	**460**	792	21	16.1	21	23.2	29	18	7	5	420	6996	-9.5
p5.2.s2t2	**510**	700	21	18.3	22	25.1	30	22	6	4	385	5379	-32.5
p5.2.s2t3	**505**	454	16	16.2	20	23.1	28	19	8	0	465	6779	-8.6
p5.3.s0	**325**	372	17	12.8	15	19.5	23	15	7	0	305	6955	-6.6
p5.3.s0t2	**350**	391	17	11.8	14	18.6	22	14	7	0	340	5230	-2.9
p5.3.s0t3	**430**	267	16	14.4	18	21.4	26	18	7	0	315	6911	-36.5
p5.3.s1	**340**	364	16	13.4	17	20.4	25	16	7	0	215	3019	-58.1
p5.3.s1t2	**355**	548	16	12.6	16	19.8	24	15	9	0	245	6766	-44.9
p5.3.s1t3	**380**	901	17	10.8	12	17.7	20	16	6	1	315	2749	-20.6
p5.3.s2	345	572	16	13.8	17	20.8	25	17	7	1	**350**	4677	1.4
p5.3.s2t2	**370**	562	16	14.8	19	21.7	27	18	7	0	350	7204	-5.7
p5.3.s2t3	**395**	842	16	13.4	16	20.5	24	17	8	1	355	5311	-11.3
p5.4.s0	340	398	17	12.0	14	18.8	22	14	7	0	**360**	4886	5.6
p5.4.s0t2	**430**	625	16	16.1	20	23.1	28	20	8	1	355	4326	-21.1
p5.4.s0t3	**405**	601	16	15.6	20	22.5	28	19	8	0	360	6408	-12.5
p5.4.s1	**330**	785	17	13.2	15	19.9	23	16	7	1	**330**	7406	0.0
p5.4.s1t2	**310**	489	17	11.5	14	18.4	22	14	6	0	275	3539	-12.7
p5.4.s1t3	**345**	574	16	13.1	17	20.2	25	17	7	0	280	1772	-23.2
p5.4.s2	**330**	523	16	14.2	19	21.3	27	18	7	1	**330**	1916	0.0
p5.4.s2t2	**360**	420	16	13.7	18	20.7	26	18	8	0	255	3597	-41.2
p5.4.s2t3	**355**	542	16	13.4	17	20.5	25	17	7	0	275	5717	-29.1
	403.9	563.2	17.0	14.5	18.1	21.4	26.0	17.5	7.3	0.9	366.1	4828.5	-13.3

Conclusions and Future Developments

Already obtained:

- an effective heuristic (EIKS) for the solution of the MCTOP
- an effective primal heuristic (PKS)
- a Branch-and-Cut algorithm outperforming plain GUROBI when solving the compact formulation in 2 hours time limit

To be done:

- extension of affinity concept to a general MIP
- final refinement for EIKS
- validation of PKS as primal heuristic in a MIP solver