

# A Two-phase Kernel Search variant for the Multidimensional Multiple-choice Knapsack Problem

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## 1. Best known results

Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH1	10	100	10	25670	<b>25670</b>	25663	25657	25664	25656
MAVCH2	10	100	10	29599	<b>29599</b>	<b>29599</b>	29598	<b>29599</b>	29598
MAVCH3	10	100	10	29180	29179	<b>29180</b>	29167	29169	29168
MAVCH4	10	100	10	30191	<b>30191</b>	30188	30178	30176	30172
MAVCH5	15	100	10	32727	<b>32727</b>	32726	32707	32720	32715
MAVCH6	15	100	10	33250	33249	<b>33250</b>	33232	33239	33234
MAVCH7	15	100	10	32238	32235	<b>32238</b>	32222	32222	32232
MAVCH8	15	100	10	29224	29223	<b>29224</b>	29207	29218	29221
MAVCH9	20	100	10	31283	31282	<b>31283</b>	31278	31275	31274
MAVCH10	20	100	10	27303	<b>27303</b>	27297	27280	27288	27290
MAVCH11	20	100	10	29288	<b>29288</b>	29284	29271	29279	29272
MAVCH12	20	100	10	30282	30280	<b>30282</b>	30262	30273	30270
MAVCH13	25	100	10	30887	<b>30887</b>	<b>30887</b>	30873	30873	30867
MAVCH14	25	100	10	40890	40885	<b>40890</b>	40869	40880	40869
MAVCH15	25	100	10	25372	25364	<b>25372</b>	25359	25354	25366
MAVCH16	25	100	10	31389	31386	<b>31389</b>	31376	31377	<b>31389</b>
MAVCH17	10	250	10	71679	<b>71679</b>	<b>71679</b>	71660	71674	71664
MAVCH18	10	250	10	72929	<b>72929</b>	<b>72929</b>	72914	72918	72924
MAVCH19	10	250	10	69207	69205	<b>69207</b>	69197	69198	69194
MAVCH20	10	250	10	60397	<b>60397</b>	<b>60397</b>	60390	60394	60393
MAVCH21	15	250	10	78153	78152	<b>78153</b>	78142	78147	78145
MAVCH22	15	250	10	81905	<b>81905</b>	81900	81895	81898	81893
MAVCH23	15	250	10	93166	93163	<b>93166</b>	93148	93151	93147
MAVCH24	15	250	10	76879	<b>76879</b>	76878	76871	76869	76864
MAVCH25	20	250	10	68242	<b>68242</b>	<b>68242</b>	68231	68234	68229
MAVCH26	20	250	10	73204	73201	<b>73204</b>	73198	73193	73195
MAVCH27	20	250	10	74471	<b>74471</b>	74470	74469	74468	74457
MAVCH28	20	250	10	50769	<b>50769</b>	<b>50769</b>	50764	50764	50766
MAVCH29	25	250	10	77241	<b>77241</b>	77238	77228	77236	77231
MAVCH30	25	250	10	67274	<b>67274</b>	67273	67259	67265	67261
MAVCH31	25	250	10	92234	<b>92234</b>	92223	92210	92217	92210
MAVCH32	25	250	10	66013	<b>66013</b>	<b>66013</b>	65996	65996	66003
MAVCH33	10	500	10	150971	150969	<b>150971</b>	150956	150960	150960
MAVCH34	10	500	10	180805	<b>180805</b>	180802	180800	180800	180786

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Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH35	10	500	10	125909	125907	<b>125909</b>	125894	125902	125903
MAVCH36	10	500	10	150887	<b>150887</b>	150886	150877	150884	150875
MAVCH37	15	500	10	176329	<b>176329</b>	176319	176316	176315	176319
MAVCH38	15	500	10	146284	<b>146284</b>	146276	146273	146277	146265
MAVCH39	15	500	10	138828	<b>138828</b>	138822	138819	138820	138812
MAVCH40	15	500	10	131316	131313	<b>131316</b>	131306	131306	131311
MAVCH41	20	500	10	136489	<b>136489</b>	136481	136483	136482	136467
MAVCH42	20	500	10	159027	<b>159027</b>	159001	159020	159021	159012
MAVCH43	20	500	10	139078	<b>139078</b>	139061	139061	139071	139053
MAVCH44	20	500	10	136521	<b>136521</b>	136501	136500	136518	136482
MAVCH45	25	500	10	91994	<b>91994</b>	<b>91994</b>	91983	91989	91988
MAVCH46	25	500	10	149554	<b>149554</b>	<b>149554</b>	149539	149543	149538
MAVCH47	25	500	10	157049	157047	157047	157025	157035	<b>157049</b>
MAVCH48	25	500	10	159504	<b>159504</b>	159502	159488	159489	159495
MAVCH49	10	700	10	207755	<b>207755</b>	<b>207755</b>	207742	207750	207741
MAVCH50	10	700	10	186818	186817	<b>186818</b>	186806	186808	186802
MAVCH51	10	700	10	225264	<b>225264</b>	<b>225264</b>	225251	225257	225261
MAVCH52	10	700	10	183321	<b>183321</b>	183310	183307	183319	183307
MAVCH53	15	700	10	183799	<b>183799</b>	183791	183784	183789	183791
MAVCH54	15	700	10	208385	208384	<b>208385</b>	208377	208383	208360
MAVCH55	15	700	10	180391	<b>180391</b>	180385	180385	180383	180374
MAVCH56	15	700	10	236460	<b>236460</b>	236457	236448	236447	236432
MAVCH57	20	700	10	201652	<b>201652</b>	201649	201644	201640	201609
MAVCH58	20	700	10	254126	<b>254126</b>	254121	254096	254110	254076
MAVCH59	20	700	10	215626	<b>215626</b>	<b>215626</b>	215604	215616	215599
MAVCH60	20	700	10	201659	<b>201659</b>	201652	201651	201651	201628
MAVCH61	25	700	10	233828	<b>233828</b>	233822	233805	233808	233812
MAVCH62	25	700	10	184901	<b>184901</b>	184900	184883	184888	184892
MAVCH63	25	700	10	226845	226838	<b>226845</b>	226813	226832	226828
MAVCH64	25	700	10	240788	<b>240788</b>	<b>240788</b>	240765	240772	240777
MAVCH65	10	100	15	42444	<b>42444</b>	42426	42397	42419	42417
MAVCH66	10	100	15	52428	52409	<b>52428</b>	52374	52392	52376
MAVCH67	10	100	15	43970	<b>43970</b>	43958	43939	43963	43959
MAVCH68	10	100	15	45896	<b>45896</b>	45881	45856	45872	45866
MAVCH69	15	100	15	43601	<b>43601</b>	43594	43577	43592	43593
MAVCH70	15	100	15	47054	<b>47054</b>	47050	46995	47021	47020
MAVCH71	15	100	15	43072	43070	<b>43072</b>	43037	43054	43044
MAVCH72	15	100	15	52522	<b>52522</b>	52493	52491	52503	52495
MAVCH73	20	100	15	53683	<b>53683</b>	53645	53604	53631	53603
MAVCH74	20	100	15	41659	<b>41659</b>	<b>41659</b>	41631	<b>41659</b>	41607
MAVCH75	20	100	15	47741	<b>47741</b>	47706	47721	47713	47687
MAVCH76	20	100	15	44638	<b>44638</b>	44604	44583	44592	44571
MAVCH77	25	100	15	51738	<b>51738</b>	51723	51693	51697	51699
MAVCH78	25	100	15	29762	<b>29762</b>	29738	29726	<b>29762</b>	29738
MAVCH79	25	100	15	45200	<b>45200</b>	45198	45178	45177	45174
MAVCH80	25	100	15	42750	42748	<b>42750</b>	42733	42712	42707
MAVCH81	10	250	15	105036	105035	<b>105036</b>	105022	105034	105014
MAVCH82	10	250	15	126207	126206	<b>126207</b>	126165	126187	126144
MAVCH83	10	250	15	107463	107454	<b>107463</b>	107437	107457	107435
MAVCH84	10	250	15	100030	<b>100030</b>	100021	100015	100010	100006
MAVCH85	15	250	15	106605	<b>106605</b>	106582	106570	106581	106573
MAVCH86	15	250	15	115377	<b>115377</b>	115351	115340	115363	115340
MAVCH87	15	250	15	120359	<b>120359</b>	120345	120309	120322	120329
MAVCH88	15	250	15	104151	<b>104151</b>	104137	104132	104133	104132
MAVCH89	20	250	15	149281	<b>149281</b>	149211	149212	149248	149229

Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH90	20	250	15	100643	<b>100643</b>	100618	100618	100625	100616
MAVCH91	20	250	15	85635	85634	<b>85635</b>	85618	85607	85622
MAVCH92	20	250	15	123031	<b>123031</b>	123024	122977	123022	122989
MAVCH93	25	250	15	113233	<b>113233</b>	113203	113172	113215	113177
MAVCH94	25	250	15	115739	<b>115739</b>	115710	115689	115699	115691
MAVCH95	25	250	15	120753	<b>120753</b>	120723	120697	120710	120718
MAVCH96	25	250	15	124441	<b>124441</b>	124414	124384	124432	124396
MAVCH97	10	500	15	197563	<b>197563</b>	197559	197542	197542	197529
MAVCH98	10	500	15	227667	<b>227667</b>	227664	227632	227647	227643
MAVCH99	10	500	15	230085	230083	<b>230085</b>	230083	230083	230083
MAVCH100	10	500	15	260000	259996	259981	259966	<b>260000</b>	259961
MAVCH101	15	500	15	188312	<b>188312</b>	188294	188274	188289	188276
MAVCH102	15	500	15	215868	<b>215868</b>	215852	215828	215856	215835
MAVCH103	15	500	15	193135	<b>193135</b>	<b>193135</b>	193111	193118	193099
MAVCH104	15	500	15	200898	<b>200898</b>	200892	200882	200868	200869
MAVCH105	20	500	15	183791	<b>183791</b>	183774	183747	183789	183767
MAVCH106	20	500	15	236240	<b>236240</b>	<b>236240</b>	236193	236214	236195
MAVCH107	20	500	15	261159	<b>261159</b>	261093	261068	261140	261077
MAVCH108	20	500	15	246155	<b>246155</b>	246141	246076	246128	246137
MAVCH109	25	500	15	219036	<b>219036</b>	219019	218975	218980	218981
MAVCH110	25	500	15	186469	<b>186469</b>	186457	186457	186440	186448
MAVCH111	25	500	15	168993	<b>168993</b>	168985	168960	168977	168970
MAVCH112	25	500	15	206542	<b>206542</b>	206520	206500	206514	206505
MAVCH113	10	700	15	290771	<b>290771</b>	290755	290736	290746	290741
MAVCH114	10	700	15	336236	<b>336236</b>	336232	336216	336234	336215
MAVCH115	10	700	15	273204	<b>273204</b>	273167	273163	273187	273157
MAVCH116	10	700	15	311754	311744	<b>311754</b>	311704	311716	311714
MAVCH117	15	700	15	309177	<b>309177</b>	309163	309147	309139	309152
MAVCH118	15	700	15	333634	<b>333634</b>	333618	333593	333632	333602
MAVCH119	15	700	15	361682	<b>361682</b>	361648	361633	361654	361640
MAVCH120	15	700	15	333601	<b>333601</b>	333583	333560	333582	333558
MAVCH121	20	700	15	271197	<b>271197</b>	271193	271161	271186	271187
MAVCH122	20	700	15	379845	<b>379845</b>	379807	379807	379793	379808
MAVCH123	20	700	15	337686	<b>337686</b>	337655	337618	337655	337628
MAVCH124	20	700	15	278250	278239	<b>278250</b>	278198	278205	278189
MAVCH125	25	700	15	317130	<b>317130</b>	317119	317090	317092	317111
MAVCH126	25	700	15	338135	<b>338135</b>	338106	338075	338101	338081
MAVCH127	25	700	15	303162	<b>303162</b>	303122	303118	303130	303127
MAVCH128	25	700	15	338079	<b>338079</b>	338056	337997	338053	338047
MAVCH129	10	100	20	44291	<b>44291</b>	44265	44223	44225	44233
MAVCH130	10	100	20	64758	<b>64758</b>	64710	64707	64703	64648
MAVCH131	10	100	20	63330	63287	<b>63330</b>	63232	63249	63207
MAVCH132	10	100	20	63373	<b>63373</b>	63359	63303	63286	63250
MAVCH133	15	100	20	62227	<b>62227</b>	62194	62205	62224	62119
MAVCH134	15	100	20	49830	<b>49830</b>	49765	49746	49788	49826
MAVCH135	15	100	20	56288	<b>56288</b>	56266	56221	56251	56212
MAVCH136	15	100	20	70683	<b>70683</b>	70648	70629	70650	70593
MAVCH137	20	100	20	60392	<b>60392</b>	60333	60303	60311	60268
MAVCH138	20	100	20	52897	<b>52897</b>	52846	52819	52858	52808
MAVCH139	20	100	20	67394	<b>67394</b>	67355	67355	67355	67304
MAVCH140	20	100	20	68339	<b>68339</b>	68325	68220	68296	68243
MAVCH141	25	100	20	68523	68511	<b>68523</b>	68425	68457	68392
MAVCH142	25	100	20	60476	<b>60476</b>	60444	60377	60417	60370
MAVCH143	25	100	20	71463	<b>71463</b>	71432	71365	71378	71309
MAVCH144	25	100	20	63471	<b>63471</b>	63393	63388	63439	63411

Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH145	10	250	20	155180	<b>155180</b>	155104	155094	155097	155028
MAVCH146	10	250	20	161764	<b>161764</b>	161652	161686	161709	161638
MAVCH147	10	250	20	151399	151397	<b>151399</b>	151382	151380	151317
MAVCH148	10	250	20	156650	<b>156650</b>	156624	156527	156601	156623
MAVCH149	15	250	20	140995	<b>140995</b>	140987	140920	140920	140976
MAVCH150	15	250	20	157333	157328	157310	157275	157326	<b>157333</b>
MAVCH151	15	250	20	148411	<b>148411</b>	148347	148355	148332	148338
MAVCH152	15	250	20	144681	<b>144681</b>	144620	144651	<b>144681</b>	144626
MAVCH153	20	250	20	131359	<b>131359</b>	131309	131268	131325	131299
MAVCH154	20	250	20	157645	<b>157645</b>	157587	157573	157586	157550
MAVCH155	20	250	20	168851	<b>168851</b>	168768	168759	168780	168739
MAVCH156	20	250	20	150128	<b>150128</b>	150058	150071	150110	150058
MAVCH157	25	250	20	139055	<b>139055</b>	139009	138994	139008	138971
MAVCH158	25	250	20	167776	<b>167776</b>	167674	167641	167698	167658
MAVCH159	25	250	20	139145	<b>139145</b>	139111	139083	139114	139056
MAVCH160	25	250	20	154051	<b>154051</b>	153994	153989	153982	153968
MAVCH161	10	500	20	285838	<b>285838</b>	285779	285768	285816	285777
MAVCH162	10	500	20	293403	<b>293403</b>	293341	293389	293367	293367
MAVCH163	10	500	20	278220	<b>278220</b>	278207	278201	<b>278220</b>	278149
MAVCH164	10	500	20	313359	<b>313359</b>	313260	313262	313291	313246
MAVCH165	15	500	20	312239	<b>312239</b>	312180	312140	312212	312190
MAVCH166	15	500	20	329599	<b>329599</b>	329526	329548	329548	329521
MAVCH167	15	500	20	297277	<b>297277</b>	297256	297234	297253	297199
MAVCH168	15	500	20	304782	304768	304724	304689	<b>304782</b>	304761
MAVCH169	20	500	20	253097	<b>253097</b>	253085	253051	253093	253085
MAVCH170	20	500	20	330342	330332	330212	<b>330342</b>	330284	330239
MAVCH171	20	500	20	267873	267857	267762	267764	<b>267873</b>	267763
MAVCH172	20	500	20	365455	<b>365455</b>	365364	365353	365360	365318
MAVCH173	25	500	20	330744	<b>330744</b>	330675	330646	330646	330642
MAVCH174	25	500	20	343423	<b>343423</b>	343315	343295	343351	343336
MAVCH175	25	500	20	335812	<b>335812</b>	335688	335696	335716	335681
MAVCH176	25	500	20	253384	<b>253384</b>	253312	253314	253335	253321
MAVCH177	10	700	20	446188	<b>446188</b>	446123	446102	446120	446129
MAVCH178	10	700	20	418041	<b>418041</b>	417982	417938	418018	417897
MAVCH179	10	700	20	446002	<b>446002</b>	445908	445924	445940	445917
MAVCH180	10	700	20	414364	<b>414364</b>	414308	414286	414342	414254
MAVCH181	15	700	20	367364	<b>367364</b>	367296	367277	367311	367276
MAVCH182	15	700	20	416396	<b>416396</b>	416351	416301	416338	416321
MAVCH183	15	700	20	514401	<b>514401</b>	514320	514315	514321	514301
MAVCH184	15	700	20	409213	<b>409213</b>	409197	409104	409199	409136
MAVCH185	20	700	20	466308	<b>466308</b>	466245	466186	466265	466179
MAVCH186	20	700	20	427838	<b>427838</b>	427752	427741	427741	427704
MAVCH187	20	700	20	420752	420726	420682	420619	<b>420752</b>	420642
MAVCH188	20	700	20	319274	<b>319274</b>	319268	319216	319261	319224
MAVCH189	25	700	20	438570	<b>438570</b>	438522	438416	438484	438490
MAVCH190	25	700	20	372526	<b>372526</b>	372413	372418	372423	372384
MAVCH191	25	700	20	407263	407261	407249	407156	<b>407263</b>	407249
MAVCH192	25	700	20	319918	<b>319918</b>	319878	319837	319857	319844
MAVCH193	10	100	25	90185	<b>90185</b>	90115	90105	90172	89961
MAVCH194	10	100	25	74166	74157	74147	74114	<b>74166</b>	73989
MAVCH195	10	100	25	62479	<b>62479</b>	62391	62424	62441	62424
MAVCH196	10	100	25	70804	<b>70804</b>	70682	70702	70767	70691
MAVCH197	15	100	25	74234	74224	<b>74234</b>	74073	74199	74025
MAVCH198	15	100	25	69184	69131	<b>69184</b>	69067	69109	68992
MAVCH199	15	100	25	62904	<b>62904</b>	62812	62828	62886	62715

Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH200	15	100	25	75424	<b>75424</b>	75338	75283	75347	75331
MAVCH201	20	100	25	74484	<b>74484</b>	<b>74484</b>	74354	74448	74289
MAVCH202	20	100	25	71433	<b>71433</b>	71297	71280	71357	71225
MAVCH203	20	100	25	81872	<b>81872</b>	81706	81779	81775	81750
MAVCH204	20	100	25	68464	<b>68464</b>	68414	68362	68422	68303
MAVCH205	25	100	25	83306	83141	83092	83072	<b>83306</b>	83082
MAVCH206	25	100	25	75690	<b>75690</b>	75581	75607	75607	75636
MAVCH207	25	100	25	78101	<b>78101</b>	78053	78020	78063	77983
MAVCH208	25	100	25	74702	<b>74702</b>	74520	74612	74595	74484
MAVCH209	10	250	25	158977	158930	158876	158839	<b>158977</b>	158881
MAVCH210	10	250	25	203152	<b>203152</b>	203010	202990	203067	202918
MAVCH211	10	250	25	177996	<b>177996</b>	177910	177872	177893	177888
MAVCH212	10	250	25	213091	<b>213091</b>	212904	213003	213057	212832
MAVCH213	15	250	25	181413	<b>181413</b>	181373	181335	181371	181259
MAVCH214	15	250	25	204921	204875	204711	204679	<b>204921</b>	204610
MAVCH215	15	250	25	171712	171686	171577	171681	171666	<b>171712</b>
MAVCH216	15	250	25	202673	<b>202673</b>	202611	202594	202594	202531
MAVCH217	20	250	25	183154	<b>183154</b>	183076	183058	183117	183048
MAVCH218	20	250	25	210642	<b>210642</b>	210524	210571	210630	210409
MAVCH219	20	250	25	199498	<b>199498</b>	199336	199426	199467	199357
MAVCH220	20	250	25	193224	193149	192933	193057	<b>193224</b>	193009
MAVCH221	25	250	25	187273	<b>187273</b>	187122	187111	187143	187078
MAVCH222	25	250	25	191169	<b>191169</b>	191027	191118	191054	190999
MAVCH223	25	250	25	212139	<b>212139</b>	211926	212006	212088	211926
MAVCH224	25	250	25	189806	<b>189806</b>	189772	189628	189722	189635
MAVCH225	10	500	25	476570	<b>476570</b>	476282	476380	476535	476255
MAVCH226	10	500	25	379380	<b>379380</b>	379278	379249	379349	379285
MAVCH227	10	500	25	349751	<b>349751</b>	349642	349532	349684	349546
MAVCH228	10	500	25	331153	331132	330894	<b>331153</b>	331123	330935
MAVCH229	15	500	25	468422	<b>468422</b>	468134	468200	468245	468261
MAVCH230	15	500	25	341039	<b>341039</b>	340925	340875	340957	340873
MAVCH231	15	500	25	385852	<b>385852</b>	385823	385688	385712	385753
MAVCH232	15	500	25	403434	<b>403434</b>	403340	403282	403376	403225
MAVCH233	20	500	25	372035	<b>372035</b>	371879	371860	371964	371903
MAVCH234	20	500	25	409289	<b>409289</b>	409056	409225	409150	409041
MAVCH235	20	500	25	339420	<b>339420</b>	339136	339326	339353	339230
MAVCH236	20	500	25	364550	<b>364550</b>	364389	364299	364407	364372
MAVCH237	25	500	25	337732	<b>337732</b>	337566	337520	337588	337550
MAVCH238	25	500	25	379967	<b>379967</b>	379812	379790	379842	379809
MAVCH239	25	500	25	370054	<b>370054</b>	369951	369804	369892	369926
MAVCH240	25	500	25	352520	<b>352520</b>	352400	352337	352419	352374
MAVCH241	10	700	25	481935	<b>481935</b>	481765	481821	481853	481819
MAVCH242	10	700	25	572655	<b>572655</b>	572550	572488	572532	572538
MAVCH243	10	700	25	506464	<b>506464</b>	506277	506276	506403	506263
MAVCH244	10	700	25	517273	<b>517273</b>	516985	517120	517227	517051
MAVCH245	15	700	25	516135	<b>516135</b>	515999	516018	516018	516011
MAVCH246	15	700	25	487918	<b>487918</b>	487854	487885	487831	487789
MAVCH247	15	700	25	446040	<b>446040</b>	445885	445882	445866	445889
MAVCH248	15	700	25	599957	<b>599957</b>	599919	599844	599890	599705
MAVCH249	20	700	25	535073	<b>535073</b>	534890	534956	534956	534879
MAVCH250	20	700	25	562844	562738	562298	562471	<b>562844</b>	562621
MAVCH251	20	700	25	524565	<b>524565</b>	524368	524259	524463	524373
MAVCH252	20	700	25	542289	<b>542289</b>	541979	542112	542183	542123
MAVCH253	25	700	25	514802	<b>514802</b>	514641	514433	514647	514590
MAVCH254	25	700	25	511299	<b>511299</b>	511117	510994	511156	511112

Instance	Items	Groups	Rss	BK	TIKS	YACA	LHBNS1	LHBNS2	MACH3
MAVCH255	25	700	25	622891	<b>622891</b>	622631	622502	622664	622626
MAVCH256	25	700	25	472663	<b>472663</b>	472556	472430	472525	472504